RADIATION OF THE GRAVITATIONAL AND ELECTROMAGNETIC BINARY PULSARS

Miroslav Pardy

INSTITUTE OF PLASMA PHYSICS ASCR
PRAGUE ASTERIX LASER SYSTEM, PALS
Za Slovankou 3, 182 21 Prague 8, Czech Republic
and
MASARYK UNIVERSITY
DEPARTMENT OF PHYSICAL ELECTRONICS
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail: pamir@physics.muni.cz

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Abstract

The energy-loss formula of the production of gravitons by the binary is derived in the source theory formulation of gravity . Then, the quantum energy loss formula involving radiative corrections is derived. We postulate an idea that gravitational pulsars are present in our universe and that radiative corrections play a role in the physics of the cosmological scale. In the last part of the article, we consider so called electromagnetic pulsar which is formed by two particles with the opposite electrical charges moving in the constant magnetic field and generating the electromagnetic pulses. We think that the cosmological analogue is possible.

1 PULSARS IN GENERAL

Pulsars are the specific cosmological objects which radiates electromagnetic energy in the form of short pulses. They were discovered by Hewish et all. in 1967, published by Hewish et all. (1968) and specified later as neutron stars. Now, it is supposed that they are fast rotating neutron stars with approximately Solar mass and with strong magnetic fields ($10^9 - 10^{14}$ Gauss). The neutron stars are formed during the evolution of stars and they are the product of the reaction $p+e^- \to n+\nu_e$, where the symbols in the last equation are as follows: proton, electron, neutron and the electron neutrino. The neutron stars were postulated by Landau (1932) at the year of the discovery of neutron in 1932. The pulsars composed from pions, or, hyperons, or, quarks probably also exist in universe, however it is not a measurement technique which rigorously determines these kinds of pulsars.

Pulsars emit highly accurate periodic signals mostly in radio waves beamed in a cone of radiation centered around their magnetic axis. These signals define the period of rotation of the neutron star, which radiates as a light-house once per revolution. We know so called slow rotated pulsars, or, normal pulsars with period P > 20ms and so called millisecond pulsars with period P < 20ms.

In 1974 a pulsar in a binary system was discovered by Hulse and Taylor and the discovery was published in 1975 (Hulse and Taylor, 1975). The period of rotation of normal pulsars increases with time and led to the rejection of the suggestion that the periodic signal could be due to the orbital period of binary stars. The orbital period of an isolated binary system decreases as it loses energy, whereas the period of a rotating body increases as it loses energy.

The present literature concerns only the electromagnetic pulsars. The number of them is 1300 and they were catalogued with very precise measurements of their positions and rotation rates. The published pulse profiles are so called integrated profiles obtained by adding some hundred of thousands of individual pulses. The integration hides a large variation of size and shape from pulse to pulse of the individual pulsar. The radiation is emitted along the direction of the field lines, so that the observed duration of the integrated profile depends on the inclination of the dipole axis to the rotation axis, because it is supposed that the pulsar radiation is a radiation of the dipole in the magnetic field. It means that the radiation of these pulsars is the synchrotron radiation of charged particles. We know that the analogical ultrarelativistic charge moving in a constant magnetic field radiates the synchrotron radiation in a very narrow cone and such system can be considered as a free electron laser in case that the opening angle of the cone is very small. Of course the angle of emission of pulsars is not smaller than radians. In other words the observed pulsars are not the free electron lasers. The idea that pulsar can be a cosmical maser was also rejected.

Pulsar radio emission is highly polarized, with linear and circular components. Individual pulses are often observed to be 100% polarized. The study of the polarization of pulsar is the starting point of the determination of their real structure.

The only energy source of the pulsar is the rotational energy of the neutron star. The rate of the dissipation of the rotational energy can be determined. The moment of inertia is fairly accurately known from the theory of the internal structure and the rotational slowdown is very accurately measured for almost every pulsar. So most of energy is radiated as magnetic dipole radiation at the rotation frequency leading to a measure of the magnetic dipole and the surface field strength.

The published articles on pulsars deal with the observation of the pulses and with the theoretical models. The observational results giving an insight into the behavior of matter in the presence of extreme gravitational and electromagnetic field are summarized for instance by Manchester (Manchester, 1992). The emission mechanism of photons are

reviewed from a plasmatical viewpoint by Melrose (1992). The morfology of the radio pulsars is presented in the recent treatise by Seiradakis et. al. (2004). The review of properties of pulsars involving the radio propagation in the magnetosphere and of emission mechanism is summarized in the article by Graham-Smith (2003). At the same time there are, to our knowledge, no information on so called gravitational pulsars, or, on the models where the pulses are produced by the retrograde motion of the bodies moving around the central body.

So, the question arises, if it is possible to define gravitational binary pulsar, where the gravitational energy is generated by the binary system, or, by the system where two components are in the retrograde motion. We suppose that in case of the massive binary system the energy is generated in a cone starting from the component of a binary and it can be seen only if the observer is present in the axis this cone. Then, the observer detects gravitational pulses when the detector is sufficiently sensitive. There are many methods for the detection of the gravitational waves. One method, based on the quantum states of the superfluid ring, was suggested by author (Pardy, 1989).

We know that gravitational waves was indirectly confirmed by the observation of the period of the pulsar PSR 1913 + 16. The energy loss of this pulsar was calculated in the framework of the classical theory of gravitation. The quantum energy loss was given for instance by Manoukian (1990). His calculation was based on the so called Schwinger source theory where gravity is considered as a field theory of gravitons where graviton is a boson with spin 2, helicity ± 2 and zero mass. It is an analogue of photon in the electromagnetic theory.

In the following text we start with the source derivation of the power spectral formula of the gravitational radiation of a binary. Then, we calculate the quantum energy loss of a binary and the gravitational power spectrum involving radiative corrections. In the last part of an article, we consider so called electromagnetic pulsar which is formed by two particles with the opposite electrical charges which move in the constant magnetic field and generate the electromagnetic pulses.

2 THE QUANTUM GRAVITY ENERGY LOOS OF A BINARY SYSTEM

2.1 Introduction

At the present time, the existence of gravitational waves is confirmed, thanks to the experimental proof of Taylor and Hulse who performed the systematic measurement of the motion of the binary with the pulsar PSR 1913+16. They found that the generalized energy-loss formula, which follows from the Einstein general theory of relativity, is in accordance with their measurement.

This success was conditioned by the fact that the binary with the pulsar PSR 1913+16 as a gigantic system of two neutron stars, emits sufficient gravitational radiation to influence the orbital motion of the binary at the observable scale.

Taylor and Hulse, working at the Arecibo radiotelescope, discovered the radiopulsar PSR 1913+16 in a binary, in 1974, and this is now considered as the best general relativistic laboratory (Taylor, 1993).

Pulsar PSR 1913+16 is the massive body of the binary system where each of the rotating pairs is 1.4 times the mass of the Sun. These neutron stars rotate around each other with a period 7.8 hours, in an orbit not much larger than the Sun's diameter. Every 59 ms, the pulsar emits a short signal that is so clear that the arrival time of a 5-min string of a set of such signals can be resolved within 15 μs.

A pulsar model based on strongly magnetized, rapidly spinning neutron stars was soon established as consistent with most of the known facts (Huguenin et al, 1968); its electrodynamical properties were studied theoretically (Gold, 1968) and shown to be plausibly capable of generating broadband radio noise detectable over interstellar distances. The binary pulsar PSR 1913+16 is now recognized as the harbinger of a new class of unusually short-period pulsars, with numerous important applications.

Because the velocities and gravitational energies in a high-mass binary pulsar system can be significantly relativistic, strong-field and radiative effects come into play. The binary pulsar PSR 1913+16 provides significant tests of gravitation beyond the weak-field, slow-motion limit (Goldreichet al., 1969; Damour et al., 1992).

The goal of this section is not to repeat the derivation of the Einstein quadrupole formula, because this has been performed many times in general relativity and also in the Schwinger source theory in the weak-field limit (Manoukian, 1990).

We show that just in the framework of the source theory it is easy to determine the quantum energy-loss formula of the binary system. The energy-loss formula can be generalized in such a way it involves also the radiative corrections.

Since the measurement of the motion of the binaries goes on, we hope that future experiments will verify the quantum version of the energy-loss formula, involving also the radiative corrections.

2.2 The source theory formulation of the problem

We show how the total quantum loss of energy caused by the production of gravitons, emitted by the binary system of two point masses moving around each other under their gravitational interaction, can be calculated in the framework of the source theory of gravity.

Source theory (Schwinger, 1970, 1973, 1976) was initially constructed to describe the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity, where the interactions are mediated by photon and graviton respectively. The source theory of gravity forms the analogue of quantum electrodynamics because, while in QED the interaction is mediated by the photon, the gravitational interaction is mediated by the graviton (Schwinger, 1976).

The source theory of gravity invented by Schwinger is linear theory. So, the question arises if it is in the coincidence with the Einstein gravity equations which are substantially nonlinear. The answer is affirmative, because the coincidence is only with the linear approximation of the Einstein theory. The experimental results of the Schwinger theory are also in harmony with experiment. The quadrupole formula of Einstein also follows from the Schwinger version.

The unification of gravity and electromagnetism is possible only in the Schwinger source theory. It is possible when, and only when it is possible the unification of forces. And this is performed in the Schwinger source theory of all interactions where force is of the Yukawa form. The problem of unification is not new. We know from the history of physics that the Ptolemy system could not be unified with the Galileo-Newton system because in the Ptolemy system it is not defined the force which is the fundamental quantity in the GN system and the primary cause of all phenomena in this system.

Einstein gravity uses the Riemann space-time where the gravity force has not the Yukawa dynamical form. The curvature of space-time is defined as the origin of all phenomena. The gravity force in the Einstein theory is in the antagonistic contradistinction with the Yukawa force in the quantum field theory and therefore it seems that QFT, QED, QCD and EL.-WEAK theory cannot be unified with the Einstein gravity.

Manoukian (1990) derivation of the Einstein quadrupole formula in the framework of the Schwinger source theory is possible because of the coincidence of the source theory with the linear limit of the Einstein theory.

Our approach is different from Manoukian method because we derive the power spectral spectral formula $P(\omega)$ of emitted gravitons with frequency ω and then using relation $(-dE/dt) = \int d\omega P(\omega)$ we determine the energy loss E. In case of the radiative correction, we derive only the power spectral formula in the general form.

The mathematical structure of $P(\omega)$ follows directly from the action W and while in the case of the gravitational radiation, the formula is composed from the tensor of energy-momentum, then, in the case of the electromagnetic radiation formula, it involves charged vector currents.

The basic formula in the source theory is the vacuum-to-vacuum amplitude (Schwinger et al., 1976):

$$\langle 0_+|0_-\rangle = e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after region of space-time, where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements, which has the simple consequence that the associated probability amplitudes multiply and the corresponding W expressions add (Schwinger et al., 1976; Dittrich, 1978).

In the flat space-time, the field of gravitons is described by the amplitude (1) with the action (Schwinger, 1970) ($c = 1$ in the following text)

$$W(T) = 4\pi G\int(dx)(dx')$$

$$\times \quad \left[T^{\mu\nu}(x)D_+(x-x')T_{\mu\nu}(x') - \frac{1}{2}T(x)D_+(x-x')T(x')\right], \tag{2}$$

where the dimensionality of $W(T)$ is the same as the dimensionality of the Planck constant $\hbar$; $T_{\mu\nu}$ is the tensor of momentum and energy. For a particle moving along the trajectory $\mathbf{x} = \mathbf{x}(t)$, it is defined by the equation (Weinberg, 1972):

$$T^{\mu\nu}(x) = \frac{p^\mu p^\nu}{E}\delta(\mathbf{x} - \mathbf{x}(t)), \tag{3}$$

where p^μ is the relativistic four-momentum of a particle with a rest mass m and

$$p^\mu = (E, \mathbf{p}) \tag{4}$$

$$p^\mu p_\mu = -m^2, \tag{5}$$

and the relativistic energy is defined by the known relation

$$E = \frac{m}{\sqrt{1-\mathbf{v}^2}}, \tag{6}$$

where $\mathbf{v}$ is the three-velocity of the moving particle.

Symbol $T(x)$ in formula (2) is defined as $T = g_{\mu\nu}T^{\mu\nu}$, and $D_+(x-x')$ is the graviton propagator whose explicit form will be determined later.

The action W is not arbitrary because it must involve the attractive force between the gravity masses while in case of the electromagnetic situation the action must involve the repulsive force between charges of the same sign. It is very surprising that such form of Lagrangians follows from the quantum definition of the vacuum to vacuum amplitude. It

was shown by Schwinger that Einstein gravity also follows from the source theory, however the method of derivation is not the integral part of the source theory because the source theory is linear and it is not clear how to establish the equivalence between linear and nonlinear theory. String theory tries to solve the problem of the unification of all forces, however, this theory is, at the present time, not predictable and works with so called extra-dimensions which was not observed. It is not clear from the viewpoint of physics, what the dimension is. It seems that many problems can be solved in the framework of the source theory.

2.3 The power spectral formula in general

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\frac{2}{\hbar}\mathrm{Im}W\right\} \stackrel{d}{=} \exp\left\{-\int dt d\omega \frac{1}{\hbar\omega}P(\omega,t)\right\}, \tag{7}$$

where the so-called power spectral function $P(\omega,t)$ has been introduced (Schwinger et al., 1976). In order to extract this spectral function from Im W, it is necessary to know the explicit form of the graviton propagator $D_+(x-x')$. The physical content of this propagator is analogous to the content of the photon propagator. It involves the gravitons property of spreading with velocity c. It means that its explicit form is just the same as that of the photon propagator. With regard to the source theory (Schwinger et al., 1976) the x-representation of $D_+(x)$ in eq. (2) is as follows:

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x-x')} D(k), \tag{8}$$

where

$$D(k) = \frac{1}{|\mathbf{k}^2| - (k^0)^2 - i\epsilon}, \tag{9}$$

which gives

$$D_+(x-x') = \frac{i}{4\pi^2}\int_0^\infty d\omega \frac{\sin\omega|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{10}$$

Now, using formulas (2), (7) and (10), we get the power spectral formula in the following form:

$$P(\omega,t) = 4\pi G\omega \int (d\mathbf{x})(d\mathbf{x}')dt' \frac{\sin\omega|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos\omega(t-t')$$

$$\times \quad \left[T^{\mu\nu}(\mathbf{x},t)T_{\mu\nu}(\mathbf{x}',t') - \frac{1}{2}g_{\mu\nu}T^{\mu\nu}(\mathbf{x},t)g_{\alpha\beta}T^{\alpha\beta}(\mathbf{x}',t')\right]. \tag{11}$$

2.4 The power spectral formula for the binary system

In the case of the binary system with masses m_1 and m_2, we suppose that they move in a uniform circular motion around their center of gravity in the xy plane, with corresponding kinematical coordinates:

$$\mathbf{x}_1(t) = r_1(\mathbf{i}\cos(\omega_0 t) + \mathbf{j}\sin(\omega_0 t)) \tag{12}$$

$$\mathbf{x}_2(t) = r_2(\mathbf{i}\cos(\omega_0 t + \pi) + \mathbf{j}\sin(\omega_0 t + \pi)) \tag{13}$$

with

$$\mathbf{v}_i(t) = d\mathbf{x}_i/dt, \quad \omega_0 = v_i/r_i, \quad v_i = |\mathbf{v}_i| \quad (i = 1,\, 2). \tag{14}$$

For the tensor of energy and momentum of the binary we have:

$$T^{\mu\nu}(x) = \frac{p_1^\mu p_1^\nu}{E_1}\delta(\mathbf{x} - \mathbf{x}_1(t)) + \frac{p_2^\mu p_2^\nu}{E_2}\delta(\mathbf{x} - \mathbf{x}_2(t)), \tag{15}$$

where we have omitted the tensor $t_{\mu\nu}^G$, which is associated with the massless, gravitational field distributed all over space and proportional to the gravitational constant G (Cho et al., 1976):

After insertion of eq. (15) into eq. (11), we get:

$$P_{total}(\omega, t) = P_1(\omega, t) + P_{12}(\omega, t) + P_2(\omega, t), \tag{16}$$

where $(t' - t = \tau)$:

$$P_1(\omega, t) = \frac{G\omega}{r_1\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega r_1 \sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)} \cos\omega\tau$$
$$\times \left(E_1^2(\omega_0^2 r_1^2 \cos\omega_0\tau - 1)^2 - \frac{m_1^4}{2E_1^2}\right), \tag{17}$$

$$P_2(\omega, t) = \frac{G\omega}{r_2\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega r_2 \sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)} \cos\omega\tau$$
$$\times \left(E_2^2(\omega_0^2 r_2^2 \cos\omega_0\tau - 1)^2 - \frac{m_2^4}{2E_2^2}\right), \tag{18}$$

$$P_{12}(\omega, t) = \frac{4G\omega}{\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin\omega[r_1^2 + r_2^2 + 2r_1r_2\cos(\omega_0\tau)]^{1/2}}{[r_1^2 + r_2^2 + 2r_1r_2\cos(\omega_0\tau)]^{1/2}} \cos\omega\tau$$
$$\times \left(E_1E_2(\omega_0^2 r_1 r_2 \cos\omega_0\tau + 1)^2 - \frac{m_1^2m_2^2}{2E_1E_2}\right). \tag{19}$$

2.5 The quantum energy loss of the binary

Using the following relations

$$\omega_0\tau = \varphi + 2\pi l, \qquad \varphi \in (-\pi, \pi), \quad l = 0,\, \pm1,\, \pm2,\, ... \tag{20}$$

$$\sum_{l=-\infty}^{l=\infty} \cos 2\pi l \frac{\omega}{\omega_0} = \sum_{l=-\infty}^{\infty} \omega_0\delta(\omega - \omega_0 l), \tag{21}$$

we get for $P_i(\omega, t)$, with ω being restricted to positive:

$$P_i(\omega, t) = \sum_{l=1}^{\infty} \delta(\omega - \omega_0 l) P_{il}(\omega, t). \tag{22}$$

Using the definition of the Bessel function $J_{2l}(z)$

$$J_{2l}(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \cos\left(z \sin\frac{\varphi}{2}\right) \cos l\varphi, \tag{23}$$

from which the derivatives and their integrals follow, we get for P_{1l} and P_{2l} the following formulas:

$$P_{il} = \frac{2G\omega}{r_i}\Big((E_i^2(1-v_i^2)^2 - \frac{m_i^4}{2E_i^2}\Big)\int_0^{2v_i l} dx\, J_{2l}(x)$$

$$+ \quad 4E_i^2(1-v_i^2)v_i^2 J'_{2l}(2v_i l) + 4E_i^2 v_i^4 J'''_{2l}(2v_i l)\Big), \quad i = 1,\, 2. \tag{24}$$

Using $r_2 = r_1 + \epsilon$, where ϵ is supposed to be small in comparison with radii r_1 and r_2, we obtain

$$[r_1^2 + r_2^2 + 2r_1 r_2 \cos\varphi]^{1/2} \approx 2a\cos\left(\frac{\varphi}{2}\right), \tag{25}$$

with

$$a = r_1\left(1 + \frac{\epsilon}{2r_1}\right). \tag{26}$$

So, instead of eq. (19) we get:

$$P_{12}(\omega, t) = \frac{2G\omega}{a\pi}\int_{-\infty}^{\infty} d\tau \frac{\sin[2\omega a\cos(\omega_0\tau/2)]}{\cos(\omega_0\tau)/2]}\cos\omega\tau$$

$$\times \quad \left(E_1 E_2(\omega_0^2 r_1 r_2\cos\omega_0\tau + 1)^2 - \frac{m_1^2 m_2^2}{2E_1E_2}\right). \tag{27}$$

Now, we can approach the evaluation of the energy-loss formula for the binary from the power spectral formulas (24) and (27). The energy loss is defined by the relation

$$-\frac{dU}{dt} = \int P(\omega)d\omega =$$

$$\int d\omega \sum_{i,l}\delta(\omega - \omega_0 l)P_{il} + \int P_{12}(\omega)d\omega = -\frac{d}{dt}(U_1 + U_2 + U_{12}). \tag{28}$$

Or,

$$-\frac{d}{dt}U_i = \int d\omega \sum_l \delta(\omega-\omega_0 l)P_{il}, \quad -\frac{d}{dt}U_{12} = \int d\omega\sum_{i,l}\delta(\omega - \omega_0 l)P_{12l}. \tag{29}$$

From Sokolov and Ternov (1983) we learn Kapteyn's formulas:

$$\sum_{l=1}^{\infty} 2lJ'_{2l}(2lv) = \frac{v}{(1-v^2)^2}, \tag{30}$$

and

$$\sum_{l=1}^{\infty} l\int_0^{2lv} J_{2l}(x)dx = \frac{v^3}{3(1-v^2)^3}. \tag{31}$$

The formula $\sum_{l=1}^{\infty} lJ'''_{2l}(2lv) = 0$ can be obtained from formula

$$\sum_{l=1}^{\infty}\frac{1}{l}J'_{2l}(2lv) = \frac{1}{2}v^2 \tag{32}$$

by its differentiation with the respect to v (Schott, 1912).

Then, after application of eqs. (30), (31) and (32) to eqs. (24) and (28), we get:

$$-\frac{dU_i}{dt} = \frac{2G\omega_0}{r_i}\left[\left(E_i^2(v_i^2-1)^2 - \frac{m_i^4}{2E_i^2}\right)\frac{v_i^3}{3(1-v_i^2)^3} - 2E_i^2 v_i^3 + 4E_i^2 v_i^4\right]. \tag{33}$$

Instead of using Kapteyn's formulas for the interference term, we will perform a direct evaluation of the energy loss of the interference term by the ω-integration in (27). So, after some elementary modification in the ω-integral, we get:

$$-\frac{dU_{12}}{dt} = \int_0^\infty P(\omega)d\omega =$$

$$A\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\omega\omega e^{-i\omega\tau}\sin[2\omega a\cos\omega_0\tau]\left[\frac{B(C\cos\omega_0\tau+1)^2-D}{\cos(\omega_0\tau/2)}\right], \qquad (34)$$

with

$$A = \frac{G}{a\pi}, \quad B = E_1E_2, \quad C = v_1v_2, \quad D = \frac{m_1^2m_2^2}{2E_1E_2}. \qquad (35)$$

Using the definition of the δ-function and its derivative, we have, instead of eq. (34), with $v = a\omega_0$:

$$-\frac{dU_{12}}{dt} = A\omega_0\pi\int_{-\infty}^{\infty} dx\,\frac{[B(C\cos x+1)^2-D]}{\cos(x/2)} \quad \times$$

$$[\delta'(x-2v\cos(x/2)) - \delta'(x+2v\cos(x/2))]\,. \qquad (36)$$

Putting

$$x - 2v\cos(x/2) = t, \qquad (37)$$

in the first δ'-term and

$$y + 2v\cos(y/2) = t, \qquad (38)$$

in the second δ'-term, we get eq. (36) in the following form:

$$-\frac{dU_{12}}{dt} = 2A\omega_0 v\pi\int_{-\infty}^{\infty} dt\delta'(t)\times$$

$$\left\{\frac{[B(C\cos x+1)^2-D]}{(x-t)(1+v\sin(x/2))} - \frac{[B(C\cos y+1)^2-D]}{(y+t)(1-v\sin(y/2))}.\right\} \qquad (39)$$

Using the known relation for a δ-function:

$$\int dtf(t)\delta'(t) = -f'(0), \qquad (40)$$

we get the energy loss formula for the synergic term in the form:

$$-\frac{dU_{12}}{dt} = -2A\omega_0 v\pi\,\frac{d}{dt}\left\{\frac{[B(C\cos x+1)^2-D]}{(x-t)(1+v\sin(x/2))} - \frac{[B(C\cos y+1)^2-D]}{(y-t)(1-v\sin(y/2))}\right\}\Bigg|_{t=0}, \qquad (41)$$

and we recommend the final calculation of the last formula to the mathematical students.

Let us remark finally that the formulas derived for the energy loss of the binary (33) and (41) describes only the binary system and therefore their sum has not the form of the Einstein quadrupole formula. The sum forms the total produced gravitational energy, and involves not only the radiation of the individual bodies of the binary, but also the interference term. The problem of the coincidence with the Einstein quadrupole formula is open.

3 THE POWER SPECTAL FORMULA IVOLVING RADIATIVE CORRECTIONS

3.1 Introduction

We here calculate the total quantum loss of energy caused by production of gravitons emitted by the binary system in the framework of the source theory of gravity for the situation with the gravitational propagator involving radiative corrections.

We know from QED that photon can exist in the virtual state as the two body system in the form of the electron positron pair. It means that the photon propagator involves the additional process:

$$\gamma \to e^{+} + e^{-} \to \gamma. \tag{42}$$

In case of the graviton radiation, the situation is analogical with the situation in QED. Instead of eq. (42) we write

$$g \to 2e^{+} + 2e^{-} \to g, \tag{43}$$

where g is graviton and number 2 is there in order to conserve spin also during the virtual process.

Equation (43) can be of course expressed in more detail:

$$g \to \gamma + \gamma \to (e^{+} + e^{-}) + (e^{+} + e^{-}) \to \gamma + \gamma \to g. \tag{44}$$

We will show that in the framework of the source theory it is easy to determine the quantum energy loss formula of the binary system both in case with the graviton propagator with radiative corrections.

We will investigate how the spectrum of the gravitational radiation is modified if we involve radiation corrections corresponding to the virtual pair production and annihilation in the graviton propagator. Our calculation is an analogue of the photon propagator with radiative corrections for production of photons by the Čerenkov mechanism (Pardy, 1994c,d).

Because the measurement of motion of the binaries goes on, we hope that the future experiments will verify the quantum version of the energy loss formula following from the source theory and that sooner or later the confirmation of this formula will be established.

3.2 The binary power spectrum with radiative corrections

According to source theory (Schwinger, 1973; Dittrich, 1978; Pardy, 1994c,d), the photon propagator in the Minkowski space-time with radiative correction is in the momentum representation of the form:

$$\tilde{D}(k) = D(k) + \delta D(k), \tag{45}$$

or,

$$\tilde{D}(k) = \frac{1}{|\mathbf{k}|^2 - (k^0)^2 - i\epsilon} + \int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{|\mathbf{k}|^2 - (k^0)^2 + \frac{M^2c^2}{\hbar^2} - i\epsilon}, \tag{46}$$

where m is mass of electron and the last term in equation (44) is derived on the virtual photon condition

$$|\mathbf{k}|^2 - (k^0)^2 = -\frac{M^2c^2}{\hbar^2}. \tag{47}$$

The weight function $a(M^2)$ has been derived in the following form (Schwinger, 1973; Dittrich, 1976):

$$a(M^2) = \frac{\alpha}{3\pi}\frac{1}{M^2}\left(1 + \frac{2m^2}{M^2}\right)\left(1 - \frac{4m^2}{M^2}\right)^{1/2}. \tag{48}$$

We suppose that the graviton propagator with the radiative corrections forms the analogue of the photon propagator.

Now, with regard to the definition of the Fourier transform

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x-x')} D(k), \tag{49}$$

we get for δD_+ the following relation ($c = \hbar = 1$):

$$\delta D_+(x-x') = \frac{i}{4\pi^2}\int_{4m^2}^{\infty} dM^2 a(M^2)$$

$$\times \int d\omega \frac{\sin\left\{[\omega^2 - M^2]^{1/2}|\mathbf{x}-\mathbf{x}'|\right\}}{|\mathbf{x}-\mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{50}$$

The function (50) differs from the gravitational function "D_+" in (9) especially by the factor

$$\left(\omega^2 - M^2\right)^{1/2} \tag{51}$$

in the function 'sin' and by the additional mass-integral which involves the radiative corrections to the original power spectrum formula.

In order to determine the additional spectral function of produced gravitons, corresponding to the radiative corrections, we insert $D_+(x-x') + \delta D_+(x-x')$ into eq. (2), and using eq. (11) we obtain (factor 2 from the two photons is involved):

$$\delta P(\omega,t) = \frac{4G\omega}{\pi}\int (d\mathbf{x})(d\mathbf{x}')dt' \int_{4m^2}^{\infty} dM^2 a(M^2)$$

$$\times \quad \frac{\sin\left\{[\omega^2 - M^2]^{1/2}|\mathbf{x}-\mathbf{x}'|\right\}}{|\mathbf{x}-\mathbf{x}'|}\cos\omega(t-t')$$

$$\times \quad \left[T^{\mu\nu}(\mathbf{x},t)g_{\mu\alpha}g_{\nu\beta}T^{\alpha\beta}(\mathbf{x}',t') - \frac{1}{2}g_{\mu\nu}T^{\mu\nu}(\mathbf{x},t)g_{\alpha\beta}T^{\alpha\beta}(\mathbf{x}',t')\right]. \tag{52}$$

Then using eqs. (16), (17), (18) and (19), we get

$$\delta P_{total}(\omega,t) = \delta P_1(\omega,t) + \delta P_2(\omega,t) + \delta P_{12}(\omega,t), \tag{53}$$

where ($t' - t = \tau$):

$$\delta P_1(\omega,t) = \frac{2G\omega}{r_1\pi}\int_{-\infty}^{\infty} d\tau \int_{4m^2}^{\infty} dM^2 a(M^2) \frac{\sin\{2\left(\omega^2 - M^2\right)^{1/2} r_1 \sin(\omega_0\tau/2)\}}{\sin(\omega_0\tau/2)} \cos\omega\tau$$

$$\times \quad \left(E_1^2(\omega_0^2 r_1^2 \cos\omega_0\tau - 1)^2 - \frac{m_1^4}{2E_1^2}\right), \tag{54}$$

$$\delta P_2(\omega,t) = \frac{2G\omega}{r_2\pi}\int_{-\infty}^{\infty} d\tau \int_{4m^2}^{\infty} dM^2 a(M^2) \frac{\sin\{2\left(\omega^2 - M^2\right)^{1/2} r_2 \sin(\omega_0\tau/2)\}}{\sin(\omega_0\tau/2)} \cos\omega\tau$$

$$\times \quad \left(E_2^2(\omega_0^2 r_2^2 \cos\omega_0\tau - 1)^2 - \frac{m_2^4}{2E_2^2}\right), \tag{55}$$

$$\delta P_{12}(\omega,t) = \frac{8G\omega}{\pi}\int_{-\infty}^{\infty} d\tau \int_{4m^2}^{\infty} dM^2 a(M^2)$$

$$\frac{\sin\{(\omega^2 - M^2)^{1/2}\left[r_1^2 + r_2^2 + 2r_1 r_2 \cos(\omega_0\tau)\right]^{1/2}\}}{\left[r_1^2 + r_2^2 + 2r_1 r_2 \cos(\omega_0\tau)\right]^{1/2}} \cos\omega\tau$$

$$\times \quad \left(E_1 E_2(\omega_0^2 r_1 r_2 \cos\omega_0\tau + 1)^2 - \frac{m_1^2 m_2^2}{2E_1 E_2}\right). \tag{56}$$

The explicit determination of the power spectrum is the problem which was solved by author in 1994. The solution was performed only approximately. Here also can be expected only the approximative solution. From this solution can be then derived the energy loss as in the previous article.

Let us show the possible way of the determination of the spectral formula. If we introduce the new variable s by the relation

$$\omega^2 - M^2 = s^2; \quad -dM^2 = 2sds \tag{57}$$

then, instead of eqs. (54), (55) and (56) we have

$$\delta P_1(\omega,t) = \frac{2G\omega}{r_1\pi}\int_{-\infty}^{\infty} d\tau \int_{s_1}^{s_2} (2sds) a(\omega^2 - s^2) \frac{\sin\{2sr_1 \sin(\omega_0\tau/2)\}}{\sin(\omega_0\tau/2)} \cos\omega\tau$$

$$\times \quad \left(E_1^2(\omega_0^2 r_1^2 \cos\omega_0\tau - 1)^2 - \frac{m_1^4}{2E_1^2}\right), \tag{58}$$

$$\delta P_2(\omega,t) = \frac{2G\omega}{r_2\pi}\int_{-\infty}^{\infty} d\tau \int_{s_1}^{s_2} (2sds) a(\omega^2 - s^2) \frac{\sin\{2sr_2 \sin(\omega_0\tau/2)\}}{\sin(\omega_0\tau/2)} \cos\omega\tau$$

$$\times \quad \left(E_2^2(\omega_0^2 r_2^2 \cos\omega_0\tau - 1)^2 - \frac{m_2^4}{2E_2^2}\right), \tag{59}$$

$$\delta P_{12}(\omega,t) = \frac{8G\omega}{\pi}\int_{-\infty}^{\infty} d\tau \int_{s_1}^{s_2} (2sds) a(\omega^2 - s^2) \frac{\sin\{2s[r_1^2 + r_2^2 + 2r_1 r_2 \cos(\omega_0\tau)]^{1/2}\}}{[r_1^2 + r_2^2 + 2r_1 r_2 \cos(\omega_0\tau)]^{1/2}}$$

$$\times \quad \cos\omega\tau \left(E_1 E_2(\omega_0^2 r_1 r_2 \cos\omega_0\tau + 1)^2 - \frac{m_1^2 m_2^2}{2E_1 E_2}\right), \tag{60}$$

where

$$s_1 = \omega^2 - 4m^2, \quad s_2 = \infty. \tag{61}$$

It seems that the rigorous procedure is the τ-integration as the first step and then s-integration as the second step (Pardy, 1994c,d). While, in case of the linear motion

the mathematical operations are easy (Pardy, 1994c), in case of the circular motion there are some difficulties (Pardy, 1994d). The final form of eq. (60) is recommended for the mathematical experts.

The energy loss is given as follows:

$$-\frac{dU_i}{dt} = \int_0^\infty d\omega P_i(\omega, t); \quad -\frac{dU_{12}}{dt} = \int_0^\infty d\omega P_{12}(\omega, t). \tag{62}$$

Now, let us go to the discussion on the electromagnetic system of the two opposite charges moving in the constant magnetic field and producing the pulse synchrotron radiation.

4 ELECTROMAGNETIC PULSAR

4.1 Introduction

Here, the power spectrum formula of the synchrotron radiation generated by the electron and positron moving at the opposite angular velocities in homogeneous magnetic field is derived in the Schwinger version of quantum field theory.

It is surprising that the spectrum depends periodically on radiation frequency ω and time which means that the system composed from electron, positron and magnetic field forms the pulsar.

We will show that the large hadron collider (LHC) which is at present time under construction in CERN can be considered in near future also as the largest electromagnetic terrestrial pulsar. We know that while the Fermilab's Tevatron handles counter-rotating protons and antiprotons in a single beam channel, LHC will operate with proton and proton beams in such a way that the collision center of mass energy will be 14 TeV and luminosity $10^{34}\mathrm{cm}^{-1}\mathrm{s}^{-2}$. To achieve such large luminosity it must operate with more than 2800 bunches per beam and a very high density of particles in bunches. The LHC will also operate for heavy Pb ion physics at a luminosity of $10^{27}\mathrm{cm}^{-1}\mathrm{s}^{-2}$ (Evans, 1999).

The collision of particles is caused by the opposite directional motion of bunches. Or, if one bunch has the angular velocity ω, then the bunch with antiparticles has angular velocity $-\omega$. Here we will determine the spectral density of emitted photons in the simplified case where one electron and one positron move in the opposite direction on a circle. We will show that the synergic spectrum depends periodically on time. This means that the behavior of the system is similar to the behavior of electromagnetic pulsar. The derived spectral formula describes the spectrum of photons generated by the Fermilab Tevatron. In case that the particles in bunches are of the same charge as in LHC, then, it is necessary to replace the function sine by cosine in the final spectral formula. Now let us approach the theory and explicit calculation of the spectrum.

This process is the generalization of the one-charge synergic synchrotron-Čerenkov radiation which has been calculated in source theory two decades ago by Schwinger et al. (1976). We will follow the Schwinger article and also the author articles (Pardy, 1994d, 2000, 2002) as the starting point. Although our final problem is the radiation of the two-charge system in vacuum, we consider, first in general, the presence of dielectric medium, which is represented by the phenomenological index of refraction n and it is well known that this phenomenological constant depends on the external magnetic field. Introducing the phenomenological constant enables to consider also the Čerenkovian processes. Later we put $n = 1$.

We will investigate here how the original Schwinger (et al.) spectral formula of the synergic c synchrotron Čerenkov radiation of the charged particle is modified if we consider the electron and positron moving at the opposite angular velocities. This problem is an

analogue of the linear (Pardy, 1997) and circular problem solved by author (Pardy, 2000). We will show that the original spectral formula of the synergic synchrotron-Čerenkov radiation is modulated by function $4\sin^2(\omega t)$ where ω is the frequency of the synergistic radiation produced by the system and it does not depend on the orbital angular frequency of electron or positron. We will use here the fundamental ingredients of Schwinger source theory (Schwinger, 1970, 1973; Dittrich, 1978; Pardy, 1994c, d, e) to determine the power spectral formula.

4.2 Formulation of the electromagnetic problem

The basic formula of the Schwinger source theory is the so called vacuum to vacuum amplitude:$\langle 0_+|0_-\rangle = \exp\{\frac{i}{\hbar}W\}$, where in case of the electromagnetic field in the medium, the action W is given by the following formula:

$$W = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{63}$$

where

$$D_+^{\mu\nu} = \frac{\mu}{c}[g^{\mu\nu} + (1-n^{-2})\beta^\mu\beta^\nu]D_+(x-x'), \tag{64}$$

where $\beta^\mu \equiv (1, \mathbf{0})$, $J^\mu \equiv (c\varrho, \mathbf{J})$ is the conserved current, μ is the magnetic permeability of the medium, ϵ is the dielectric constant od the medium and $n = \sqrt{\epsilon\mu}$ is the index of refraction of the medium. Function D_+ is defined as in eq. (10) (Schwinger et al., 1976):

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{65}$$

The probability of the persistence of vacuum follows from the vacuum amplitude (1) where Im W is the basis for the following definition of the spectral function $P(\omega, t)$:

$$-\frac{2}{\hbar}\mathrm{Im}\, W \stackrel{d}{=} -\int dtd\omega\frac{P(\omega,t)}{\hbar\omega}. \tag{66}$$

Now, if we insert eq. (64) into eq. (66), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\left[\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\right]$$

$$\times \quad \cos[\omega(t-t')][\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')], \tag{67}$$

which is an analogue of the formula (11).

Let us recall that the last formula can be derived also in the classical electrodynamic context as it is shown for instance in the Schwinger article (Schwinger, 1949). The derivation of the power spectral formula from the vacuum amplitude is more simple.

4.3 The radiation of two opposite charges

Now, we will apply the formula (67) to the two-body system with the opposite charges moving at the opposite angular velocities in order to get in general synergic synchrotron-Čerenkov radiation of electron and positron moving in a uniform magnetic field

While the synchrotron radiation is generated in a vacuum, the synergic synchrotron-Čerenkov radiation can produced only in a medium with dielectric constant n. We suppose

the circular motion with velocity $\mathbf{v}$ in the plane perpendicular to the direction of the constant magnetic field $\mathbf{H}$ (chosen to be in the $+z$ direction).

We can write the following formulas for the charge density ϱ and for the current density $\mathbf{J}$ of the two-body system with opposite charges and opposite angular velocities:

$$\varrho(\mathbf{x}, t) = e\delta(\mathbf{x} - \mathbf{x_1}(t)) - e\delta(\mathbf{x} - \mathbf{x_2}(t)) \tag{68}$$

and

$$\mathbf{J}(\mathbf{x}, t) = e\mathbf{v}_1(t)\delta(\mathbf{x} - \mathbf{x_1}(t)) - e\mathbf{v}_2(t)\delta(\mathbf{x} - \mathbf{x_2}(t)) \tag{69}$$

with

$$\mathbf{x}_1(t) = \mathbf{x}(t) = R(\mathbf{i}\cos(\omega_0 t) + \mathbf{j}\sin(\omega_0 t)), \tag{70}$$

$$\mathbf{x}_2(t) = R(\mathbf{i}\cos(-\omega_0 t) + \mathbf{j}\sin(-\omega_0 t) = \mathbf{x}(-\omega_0, t) = \mathbf{x}(-t). \tag{71}$$

The absolute values of velocities of both particles are the same, or $|\mathbf{v}_1(t)| = |\mathbf{v}_2(t)| = v$, where ($H = |\mathbf{H}|, E =$ energy of a particle)

$$\mathbf{v}(t) = d\mathbf{x}/dt, \quad \omega_0 = v/R, \quad R = \frac{\beta E}{eH}, \quad \beta = v/c, \quad v = |\mathbf{v}|. \tag{72}$$

After insertion of eqs. (68)–(71) into eq. (67), and after some mathematical operations we get

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}e^2\int_{-\infty}^{\infty} dt' \cos(t - t')\sum_{i,j=1}^{2}(-1)^{i+j}$$
$$\times \quad \left[1 - \frac{\mathbf{v}_i(t)\cdot\mathbf{v}_j(t')}{c^2}n^2\right]\left\{\frac{\sin\frac{n\omega}{c}|\mathbf{x}_i(t) - \mathbf{x}_j(t')|}{|\mathbf{x}_i(t) - \mathbf{x}_j(t')|}\right\}. \tag{73}$$

Let us remark, that for situation of the identical charges, the factor $(-1)^{i+j}$ must be replaced by 1.

Using $t' = t + \tau$, we get for

$$\mathbf{x}_i(t) - \mathbf{x}_j(t') \stackrel{d}{=} \mathbf{A}_{ij}, \tag{74}$$

$$|\mathbf{A}_{ij}| = [R^2 + R^2 - 2RR\cos(\omega_0\tau + \alpha_{ij})]^{1/2} = 2R\left|\sin\left(\frac{\omega_0\tau + \alpha_{ij}}{2}\right)\right|, \tag{75}$$

where α_{ij} were evaluated as follows:

$$\alpha_{11} = 0, \quad \alpha_{12} = 2\omega_0 t, \quad \alpha_{21} = 2\omega_0 t, \quad \alpha_{22} = 0. \tag{76}$$

Using

$$\mathbf{v}_i(t)\cdot\mathbf{v}_j(t + \tau) = \omega_0^2 R^2\cos(\omega_0\tau + \alpha_{ij}), \tag{77}$$

and relation (75) we get with $v = \omega_0 R$

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}e^2\int_{-\infty}^{\infty} d\tau \cos\omega\tau\sum_{i,j=1}^{2}(-1)^{i+j}$$
$$\times \quad \left[1 - \frac{n^2}{c^2}v^2\cos(\omega_0\tau + \alpha_{ij})\right]\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{(\omega_0\tau + \alpha_{ij})}{2}\right)\right]}{2R\sin\left(\frac{(\omega_0\tau + \alpha_{ij})}{2}\right)}\right\}. \tag{78}$$

Introducing new variable T by relation

$$\omega_0\tau + \alpha_{ij} = \omega_0 T \tag{79}$$

for every integral in eq. (78), we get $P(\omega,t)$ in the following form

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{e^2}{2R}\frac{\mu}{n^2}\int_{-\infty}^{\infty} dT \sum_{i,j=1}^{2}(-1)^{i+j}$$

$$\times \quad \cos(\omega T - \frac{\omega}{\omega_0}\alpha_{ij})\left[1 - \frac{c^2}{n^2}v^2\cos(\omega_0 T\right]\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}. \tag{80}$$

The last formula can be written in the more compact form,

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\frac{e^2}{2R}\sum_{i,j=1}^{2}(-1)^{i+j}\left\{P_1^{(ij)} - \frac{n^2}{c^2}v^2 P_2^{(ij)}\right\}, \tag{81}$$

where

$$P^{(ij)} = J_{1a}^{(ij)}\cos\frac{\omega}{\omega_0}\alpha_{ij} + J_{1b}^{(ij)}\sin\frac{\omega}{\omega_0}\alpha_{ij} \tag{82}$$

and

$$P_2^{(ij)} = J_{2A}^{(ij)}\cos\frac{\omega}{\omega_0}\alpha_{ij} + J_{2B}^{(ij)}\sin\frac{\omega}{\omega_0}\alpha_{ij}, \tag{83}$$

where

$$J_{1a}^{(ij)} = \int_{-\infty}^{\infty} dT\cos\omega T\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}, \tag{84}$$

$$J_{1b}^{(ij)} = \int_{-\infty}^{\infty} dT\sin\omega T\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}, \tag{85}$$

$$J_{2A}^{(ij)} = \int_{-\infty}^{\infty} dT\cos\omega_0 T\cos\omega T\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}, \tag{86}$$

$$J_{2B}^{(ij)} = \int_{-\infty}^{\infty} dT\cos\omega_0 T\sin\omega T\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}, \tag{87}$$

Using

$$\omega_0 T = \varphi + 2\pi l, \qquad \varphi \in (-\pi,\pi), \qquad l = 0,\, \pm 1,\, \pm 2,\, ..., \tag{88}$$

we can transform the T-integral into the sum of the telescopic integrals according to the scheme:

$$\int_{-\infty}^{\infty} dT \quad \longrightarrow \quad \frac{1}{\omega_0}\sum_{l=-\infty}^{l=\infty}\int_{-\pi}^{\pi} d\varphi. \tag{89}$$

Using the fact that for the odd functions $f(\varphi)$ and $g(l)$, the relations are valid

$$\int_{-\pi}^{\pi} f(\varphi)d\varphi = 0; \quad \sum_{l=-\infty}^{l=\infty} g(l) = 0, \tag{90}$$

we can write

$$J_{1a}^{(ij)} = \frac{1}{\omega_0}\sum_l \int_{-\pi}^{\pi} d\varphi \left\{\cos\frac{\omega}{\omega_0}\varphi\cos 2\pi l\frac{\omega}{\omega_0}\right\}\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\varphi}{2}\right)\right]}{\sin\left(\frac{\varphi}{2}\right)}\right\}, \tag{91}$$

$$J_{1b}^{(ij)} = 0. \tag{92}$$

For integrals with indices A, B we get:

$$J_{2A}^{(ij)} = \frac{1}{\omega_0}\sum_l \int_{-\pi}^{\pi} d\varphi\cos\varphi \left\{\cos\frac{\omega}{\omega_0}\varphi\cos 2\pi l\frac{\omega}{\omega_0}\right\}\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\varphi}{2}\right)\right]}{\sin\left(\frac{\varphi}{2}\right)}\right\}, \tag{93}$$

$$J_{2B}^{(ij)} = 0, \tag{94}$$

So, the power spectral formula (80) is of the form:

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\frac{e^2}{2R}\sum_{i,j=1}^{2}(-1)^{i+j}\left\{P_1^{(ij)} - n^2\beta^2 P_2^{(ij)}\right\}; \quad \beta = \frac{v}{c}, \tag{95}$$

where

$$P_1^{(ij)} = J_{1a}^{(ij)}\cos\frac{\omega}{\omega_0}\alpha_{ij} \tag{96}$$

and

$$P_2^{(ij)} = J_{2A}^{(ij)}\cos\frac{\omega}{\omega_0}\alpha_{ij}. \tag{97}$$

Using the Poisson theorem

$$\sum_{l=-\infty}^{\infty}\cos 2\pi\frac{\omega}{\omega_0}l = \sum_{k=-\infty}^{\infty}\omega_0\delta(\omega - \omega_0 l), \tag{98}$$

the definition of the Bessel functions J_{2l} and their corresponding derivations and integrals

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi\cos\left(z\sin\frac{\varphi}{2}\right)\cos l\varphi = J_{2l}(z), \tag{99}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi\sin\left(z\sin\frac{\varphi}{2}\right)\sin(\varphi/2)\cos l\varphi = -J'_{2l}(z), \tag{100}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi\frac{\sin\left(z\sin\frac{\varphi}{2}\right)}{\sin(\varphi/2)}\cos l\varphi = \int_0^z J_{2l}(x)dx, \tag{101}$$

and using equations

$$\sum_{i,j=1}^{2}(-1)^{i+j}\cos\frac{\omega}{\omega_0}\alpha_{ij} = 2(1-\cos 2\omega t) = 4\sin^2\omega t, \tag{102}$$

we get with the definition of the partial power spectrum P_l

$$P(\omega) = \sum_{l=1}^{\infty}\delta(\omega - l\omega_0)P_l, \tag{103}$$

the following final form of the partial power spectrum generated by motion of two-charge system moving in the cyclotron:

$$P_l(\omega,t) = [4(\sin\omega t)^2]\frac{e^2}{\pi n^2}\frac{\omega\mu\omega_0}{v}\left(2n^2\beta^2 J'_{2l}(2ln\beta) - (1-n^2\beta^2)\int_0^{2ln\beta} dx J_{2l}(x)\right). \quad (104)$$

So we see that the spectrum generated by the system of electron and positron is formed in such a way that the original synchrotron spectrum generated by electron is modulated by function $4\sin^2(\omega t)$. The derived formula involves also the synergic process composed from the synchrotron radiation and the Čerenkov radiation for electron velocity $v > c/n$ in a medium.

Our goal is to apply the last formula in situation where there is a vacuum. In this case we can put $\mu = 1, n = 1$ in the last formula and so we have

$$P_l(\omega,t) = 4\sin^2(\omega t)\,\frac{e^2}{\pi}\frac{\omega\omega_0}{v}\left(2\beta^2 J'_{2l}(2l\beta) - (1-\beta^2)\int_0^{2l\beta} dx J_{2l}(x)\right). \quad (105)$$

So, we see, that final formula describing the opposite motion of electron and positron in accelerator is of the form

$$P_{l,pair}(\omega,t) = 4\sin^2(\omega t)\, P_{l(electron)}(\omega), \quad (106)$$

where $P_{electron}$ is the spectrum of radiation only of electron. For the same charges it is necessary to replace sine by cosine in the final formula.

The result (106)is surprising because we naively expected that the total radiation of the opposite charges should be

$$P_l(\omega,t) = P_{l(electron)}(\omega,t) + P_{l(positron)}(\omega,t). \quad (107)$$

So, we see that the resulting radiation can not be considered as generated by the isolated particles but by a synergic production of a system of particles and magnetic field. At the same time we cannot interpret the result as a result of interference of two sources because the distance between sources radically changes and so, the condition of an interference is not fulfilled.

The classical electrodynamics formula (106) changes our naive opinion on the electrodynamic processes in the magnetic field. From the last formula it follows that at time $t = \pi k/\omega$ there is no radiation of the frequency ω. The spectrum oscillates with frequency ω. If the radiation were generated not in the synergic way, then the spectral formula would be composed from two parts corresponding to two isolated sources.

5 The two center circular motions

The situation which we have analyzed was the ideal situation where the angle of collision of positron and electron was equal to π. Now, the question arises what is the modification of a spectral formula when the collision angle between particles differs from π. It can be easily seen that if the second particle follows the shifted circle trajectory, then the collision angle differs from π. Let us suppose that the center of the circular trajectory of the second particle has coordinates $(a, 0)$. It can be easy to see from the geometry of the situation and from the plane geometry that the collision angle is $\pi - \alpha, \alpha \approx \tan\alpha \approx a/R$ where R is a radius of the first or second circle. The same result follows from the analytical geometry of the situation.

While the equation of the first particle is the equation of the original trajectory, or this is eq. (70)

$$\mathbf{x}_1(t) = \mathbf{x}(t) = R(\mathbf{i}\cos(\omega_0 t) + \mathbf{j}\sin(\omega_0 t)), \tag{108}$$

the equation of a circle with a shifted center is as follows:

$$\mathbf{x}_2(t) = \mathbf{x}(t) = R(\mathbf{i}(\frac{a}{R} + \cos(-\omega_0 t)) + \mathbf{j}\sin(-\omega_0 t)) = \mathbf{x}(-\mathbf{t}) + \mathbf{i}a. \tag{109}$$

The absolute values of velocities of both particles are equal and the relation (72) is valid. Instead of equation (74) we have for radius vectors of particle trajectories:

$$\mathbf{x}_i(t) - \mathbf{x}_j(t') \stackrel{d}{=} \mathbf{B}_{ij}, \tag{110}$$

where $\mathbf{B}_{11} = \mathbf{A}_{11}, \mathbf{B}_{12} = \mathbf{A}_{12} - \mathbf{i}a, \mathbf{B}_{21} = \mathbf{A}_{21} + \mathbf{i}a, \mathbf{B}_{22} = \mathbf{A}_{22}$.

In general, we can write the last information on coefficients $\mathbf{B}_{ij}$ as follows:

$$\mathbf{B}_{ij} = \mathbf{A}_{ij} + \varepsilon_{ij}\mathbf{i}a, \tag{111}$$

where $\varepsilon_{11} = 0, \varepsilon_{12} = -1, \varepsilon_{21} = 1, \varepsilon_{22} = 0$.

For motion of particles along trajectories the absolute value of vector $\mathbf{A}_{ij} \gg a$ during the most part of the trajectory. It means, we can determine B_{ij} approximatively. After elementary operations, we get:

$$|\mathbf{B_{ij}}| = (A_{ij}^2 + 2|\mathbf{A}_{ij}|\varepsilon_{ij} a\cos\varphi_{ij} + a^2\varepsilon_{ij}^2)^{1/2}, \tag{112}$$

where $\cos\varphi_{ij}$ can be expressed by the x-component of vector $\mathbf{A}_{ij}$ and $|\mathbf{A}_{ij}|$ as follows:

$$\cos\varphi_{ij} = \frac{(A_{ij})_x}{|\mathbf{A}_{ij}|}. \tag{113}$$

After elementary trigonometric operations, we derive the following formula for $(A_{ij})_x$:

$$(A_{ij})_x = 2R\sin\frac{2\omega_0 t + \omega_0\tau}{2}\sin\frac{\omega_0\tau}{2}. \tag{114}$$

Then, using equation (114), we get with $\varepsilon = a/R$

$$|\mathbf{B_{ij}}| = 2R(\sin^2\frac{\omega_0\tau + \alpha_{ij}}{2} + \varepsilon\varepsilon_{ij}\sin\frac{2\omega_0 t + \omega_0\tau}{2}\sin\frac{\omega_0\tau}{2} + \varepsilon^2\frac{\varepsilon_{ij}^2}{4})^{1/2}. \tag{115}$$

In order to perform the τ-integration the substitution must be introduced. However, the substitution $\omega_0\tau + \alpha_{ij} = \omega_0 T$ does not work. So we define the substitution $\tau = \tau(T)$ by the following transcendental equation (we neglect the term with ε^2):

$$\left[\sin^2\frac{\omega_0\tau + \alpha_{ij}}{2} + \varepsilon\varepsilon_{ij}\sin\frac{\omega_0 t + \omega_0\tau}{2}\sin\frac{\omega_0\tau}{2}\right]^{1/2} = \sin\frac{\omega_0 T}{2}. \tag{116}$$

Or, after some trigonometrical modifications and using the approximative formula $(1+x)^{1/2} \approx 1 + x/2$ for $x \ll 1$

$$[./.]^{1/2} \approx \sin\left(\frac{\omega_0\tau + \alpha_{ij}}{2}\right) + \frac{\varepsilon}{2}\varepsilon_{ij}\sin\left(\frac{2\omega_0\tau + 2\omega_0 t - \alpha_{ij}}{2}\right) = \sin\frac{\omega_0 T}{2}. \tag{117}$$

We se that for $\varepsilon = 0$ the substitution is $\omega_0\tau + \alpha_{ij} = \omega_0 T$. The equation (117) is the transcendental equation and the exact solution is the function $\tau = \tau(T)$. We are looking for the solution of equation (117) in the approximative form using the approximation $\sin x \approx x$.

Then, instead of (117) we have:

$$\left(\frac{\omega_0\tau+\alpha_{ij}}{2}\right)+\frac{\varepsilon}{2}\varepsilon_{ij}\left(\frac{2\omega_0\tau+2\omega_0 t-\alpha_{ij}}{2}\right)=\frac{\omega_0 T}{2} \tag{118}$$

Using substitution

$$\omega_0\tau+\alpha_{ij}=\omega_0 T+\omega_0\varepsilon A \tag{119}$$

in eq. (118) we get, to the first order in ε-term:

$$A=-\frac{\varepsilon_{ij}}{2\omega_0}(\omega_0 T-2\alpha_{ij}+2\omega_0 t). \tag{120}$$

Then, after some algebraic manipulation we get:

$$\omega_0\tau+\alpha_{ij}=\omega_0 T(1-\frac{\varepsilon}{2}\varepsilon_{ij})-\varepsilon\varepsilon_{ij}\omega_0 t(-1)^{i+j} \tag{121}$$

and

$$\omega\tau=\omega T(1-\frac{\varepsilon}{2}\varepsilon_{ij})-\frac{\omega}{\omega_0}\left(\varepsilon\varepsilon_{ij}(-1)^{i+j}\omega_0 t+\alpha_{ij}\right). \tag{122}$$

For small time t, we can write approximately:

$$\cos(\omega_0\tau+\alpha_{ij})\approx\cos\omega_0 T(1-\frac{\varepsilon}{2}\varepsilon_{ij}) \tag{123}$$

and from eq. (122)

$$d\tau=dT(1-\frac{\varepsilon}{2}\varepsilon_{ij}). \tag{124}$$

So, in case of the eccentric circles the formula (118) can be obtained from non-perturbative formula (80) only by transformation

$$T\quad\longrightarrow\quad T(1-\frac{\varepsilon}{2}\varepsilon_{ij});\quad \alpha_{ij}\quad\longrightarrow\quad\left(\varepsilon\varepsilon_{ij}(-1)^{i+j}\omega_0 t+\alpha_{ij}\right)=\tilde{\alpha}_{ij}, \tag{125}$$

excepting specific term involving sine functions.

Then, instead of formula (80) we get:

$$P(\omega,t)=-\frac{\omega}{4\pi^2}\frac{e^2}{2R}\frac{\mu}{n^2}\int_{-\infty}^{\infty}dT\sum_{i,j=1}^{2}(-1)^{i+j}$$
$$\times\quad\cos(\omega T-\frac{\omega}{\omega_0}\tilde{\alpha}_{ij})\left[1-\frac{c^2}{n^2}v^2\cos(\omega_0 T)\right]\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0\tilde{T}}{2}\right)\right]}{\sin\left(\frac{\omega_0\tilde{T}}{2}\right)}\right\}. \tag{126}$$

where $\tilde{T}=T(1-\frac{\varepsilon}{2}\varepsilon_{ij})$. We see that only $\tilde{T}$ and the α term are the new modification of the original formula (80).

However, because ε term in the sine functions is of very small influence on the behavior of the total function for finite time t, we can neglect it and write approximatively:

$$P(\omega,t)=-\frac{\omega}{4\pi^2}\frac{e^2}{2R}\frac{\mu}{n^2}\int_{-\infty}^{\infty}dT\sum_{i,j=1}^{2}(-1)^{i+j}$$
$$\times\quad\cos(\omega T-\frac{\omega}{\omega_0}\tilde{\alpha}_{ij})\left[1-\frac{c^2}{n^2}v^2\cos(\omega_0 T)\right]\left\{\frac{\sin\left[\frac{2Rn\omega}{c}\sin\left(\frac{\omega_0 T}{2}\right)\right]}{\sin\left(\frac{\omega_0 T}{2}\right)}\right\}. \tag{127}$$

So, we se that only difference with the original radiation formula is in variable $\tilde{\alpha}_{ij}$.

It means that instead of sum (102) we have the following sum:

$$\sum_{i,j=1}^{2}(-1)^{i+j}\cos\frac{\omega}{\omega_0}\tilde{\alpha}_{ij} = 2(1-\cos 2\omega t\cos\varepsilon\omega t). \tag{128}$$

It means that the one electron radiation formula is not modulated by $[\sin\omega t]^2$ but by the formula (128) and the final formula of for the power spectrum is as follows:

$$P_l(\omega,t) = 2(1-\cos 2\omega t\cos\varepsilon\omega t)P_{l(electron)}(\omega)\,. \tag{129}$$

For $\varepsilon \to 0$, we get the original formula (106).

6 SUMMARY AND DISCUSSION

We have derived, in the first part of the article, the total quantum loss of energy of the binary. The energy loss is caused by the emission of gravitons during the motion of the two binary bodies around each other under their gravitational interaction. The energy-loss formulas of the production of gravitons are derived here in the source theory. It is evident that the production of gravitons by the binary system is not homogenous and isotropical in space. So, the binary forms the "gravitational light house" where instead of the light photons of the electromagnetic pulsar are the gravitons. The detector of the gravitational waves evidently detects the gravitational pulses.

This section is an extended and revised version of the older author's article (Pardy, 1983a) and preprints (Pardy,1994a,b), in which only the spectral formulas were derived. Here, in the first part of the article, we have derived the quantum energy-loss formulas for the linear gravitational field. Linear field corresponds to the weak field limit of the Einstein gravity.

The power spectrum formulas involving radiative corrections are derived in the following part of this article, also in the framework of the source theory. The general relativity necessarily does not contain the method how to express the quantum effects together with the radiative corrections by the geometrical language. So, it cannot give the answer on the production of gravitons and on the graviton propagator with radiative corrections. This section therefore deals with the quantum energy loss caused by the production of gravitons and by the radiative corrections in the graviton propagator in case of the motion of a binary.

We believe the situation in the gravity problems with radiative corrections is similar to the QED situation many years ago when the QED radiative corrections were theoretically predicted and then experimentally confirmed for instance in case of he Lamb shift, or, of the anomalous magnetic moment of electron.

Astrophysics is, in a crucial position in proving the influence of radiative corrections on the dynamics in the cosmic space. We hope that the further astrophysical observations will confirm the quantum version of the energy loss of the binary with graviton propagator with radiative corrections.

In the last part of this article on pulsars we have derived the power spectrum formula of the synchrotron radiation generated by the electron and positron moving at the opposite angular velocities in homogeneous magnetic field. It forms an analogue of the author article (Pardy, 1997) where only comoving electrons, or positrons was considered, and it forms the modified author preprints (Pardy, 2000a; 2001) and articles (Pardy, 200b; 2002,) where the power spectrum is calculated for two charges performing the retrograde motion in a magnetic field. The frequency of motion was the same because the diameter of the circle was considered the same for both charges. The retrograde motion with different diameters was not considered.

It is surprising that the spectrum depends periodically on radiation frequency ω and time which means that the system composed from electron, positron and magnetic field behaves as a pulsating system. While such pulsar can be represented by a terrestrial experimental arrangement it is possible to consider also the cosmological existence in some modified conditions.

To our knowledge, our result is not involved in the classical monographs on the electromagnetic theory and at the same time it was not still studied by the accelerator experts investigating the synchrotron radiation of bunches. This effect was not described in textbooks on classical electromagnetic field and on the synchrotron radiation. We hope that sooner or later this effect will be verified by the accelerator physicists. The radiative corrections obviously influence the synergistic spectrum of photons (Pardy, 1994c,d).

The particle laboratories used instead of the single electron and positron the bunches with 10^{10} electrons or positrons in one bunch of volume 300μm $\times$ 40μm $\times$ 0.01 m. So, in some approximation we can replace the charge of electron and positron by the charges Q and -Q of both bunches in order to get the realistic intensity of photons. Nevertheless the synergic character of the radiation of two bunches moving at the opposite direction in a magnetic field is conserved.

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